Implicit Formulation for SPH-based Viscous Fluids

Tetsuya Takahashi[1,2] Yoshinori Dobashi[3,2] Issei Fujishiro[4] Tomoyuki Nishita[2,5] Ming C. Lin[1]

[1]University of North Carolina at Chapel Hill, [2]UEI Research, [3]Hokkaido University, [4]Keio University, [5]Hiroshima Shudo University

UNC DEPARTMENT OF COMPUTER SCIENCE

http://gamma.cs.unc.edu/ViscousSPH/

CV²R Consortium for Vision and Virtual Reality

Motivation

Viscous fluids are common materials as we see their examples, such as honey, caramel sauce, melted chocolate, machinery oils, bodily fluids, in our daily lives, and thus simulating them has been required in various fields including movies, video games, and virtual simulators.

Problems

Previously proposed particle-based, Smoothed Particle Hydrodynamics (SPH) methods assume that fluid is inviscid or slightly viscous, and thus an effective SPH method has not yet been established. There are two main reasons:

- Previous SPH methods use Laplacian form of viscosity by dropping the off-diagonal component of the unsimplified, full form of viscosity, and consequently fail to handle variable viscosity and generate rotational viscous fluid behaviors, such as coiling and buckling.
- Previous SPH methods adopt explicit viscosity integration, which severely restricts time steps, making it infeasible to simulate viscous fluids within a practical time.

Method

We discretize the fluid volume using SPH and solve the Navier-Stokes equations. To address the two problems, we solve the full form of viscosity using implicit integration:

$$\boldsymbol{u}_i^{t+1} = \boldsymbol{u}_i + \frac{\Delta t}{\rho_i}\nabla\cdot\boldsymbol{s}_i^{t+1},$$

$$\boldsymbol{s}_i^{t+1} = \mu_i\left(\nabla\boldsymbol{u}_i^{t+1} + \left(\nabla\boldsymbol{u}_i^{t+1}\right)^T\right),$$

where $\boldsymbol{u}$: velocity, t: time, i: particle index, Δt: time step, ρ: density, $\boldsymbol{s}$: viscous stress tensor, and μ: dynamic viscosity. Consequently, we obtain a linear system from the implicit formulation as

$$\boldsymbol{C}\boldsymbol{U}^{t+1} = \boldsymbol{U},$$

where $\boldsymbol{C}$: coefficient matrix, $\boldsymbol{U}$: concatenation of velocities. The resulting linear system is solved with a specialized sparse matrix structure and a conjugate gradient solver.

Contributions

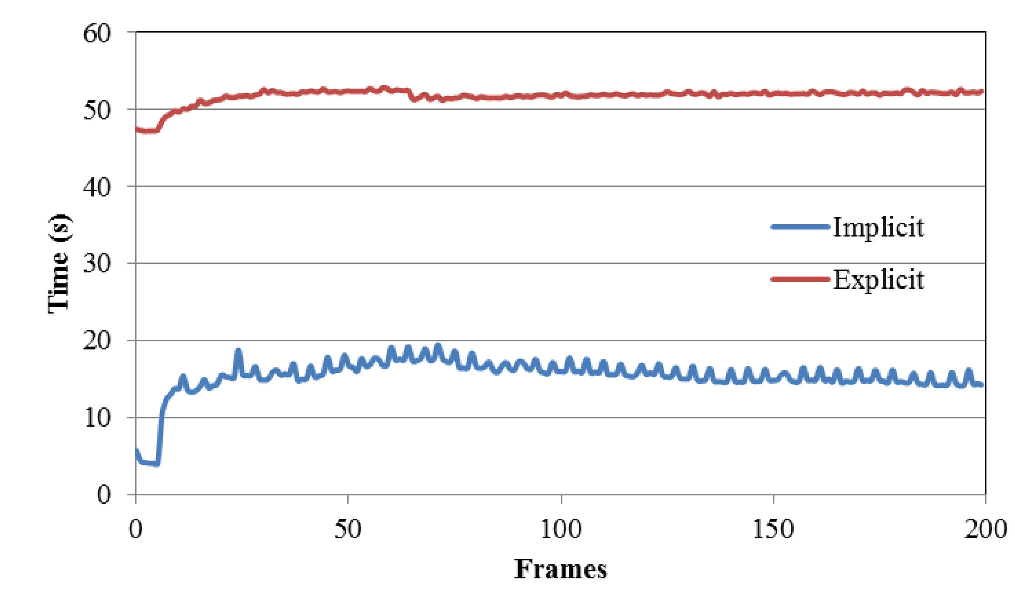
Our method offers the following advantages:

- It is efficient and robust with larger time steps and higher viscosity because of implicit integration.
- It can accurately handle variable viscosity and plausibly generate coiling and buckling phenomena by solving the full form of viscosity.

Variable Viscosity

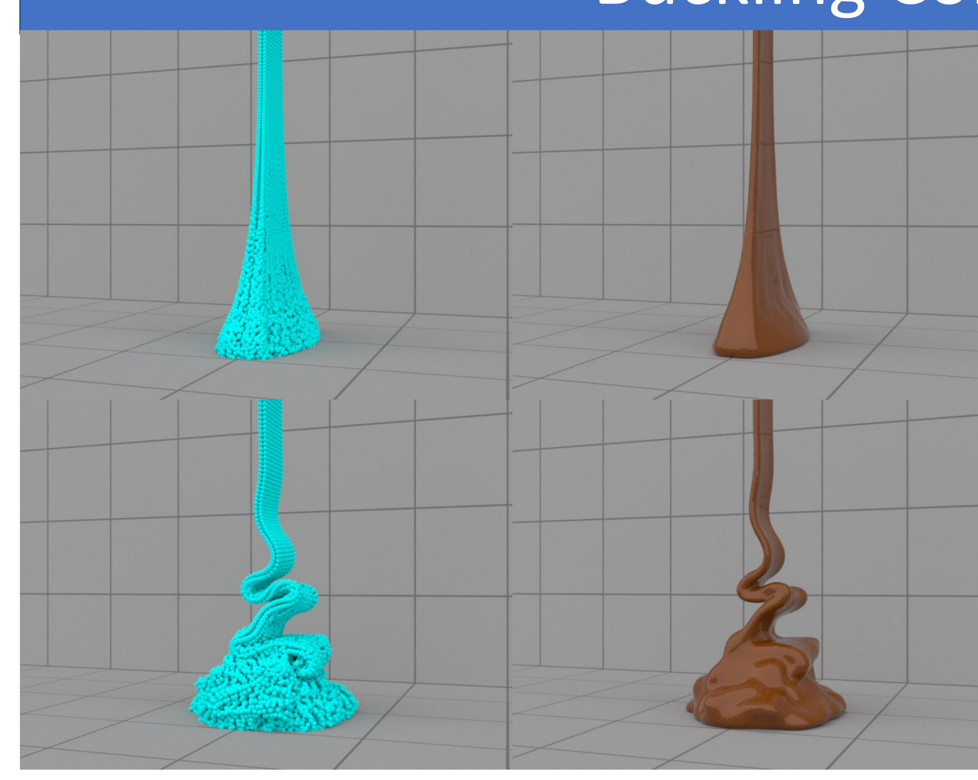
Viscous dragon with spatially varying viscosity. Our method can accurately handle variable viscosity.

Performance



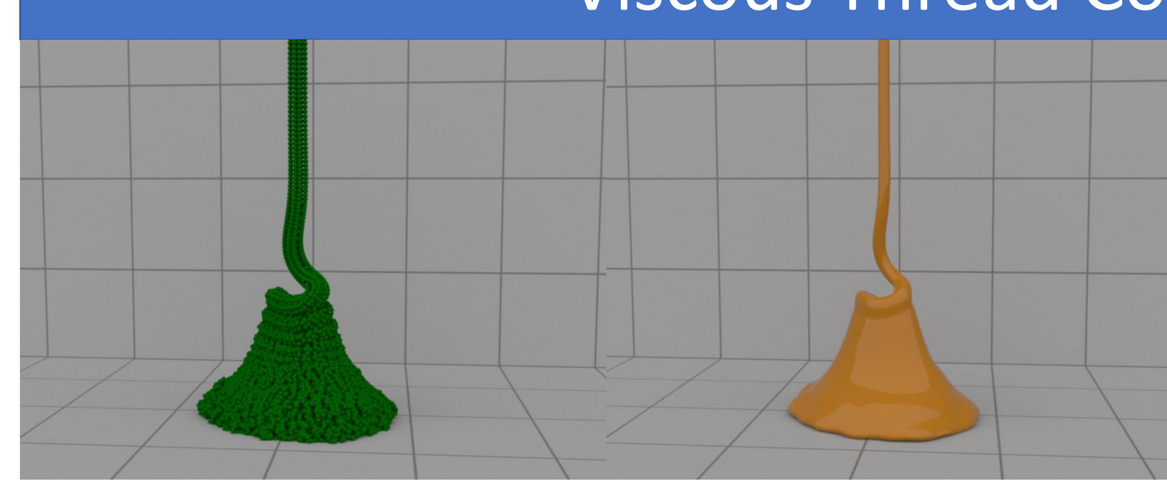
Performance profile for explicit integration and implicit integration. Implicit integration outperforms explicit integration by a factor of 3.4.

Buckling Comparison



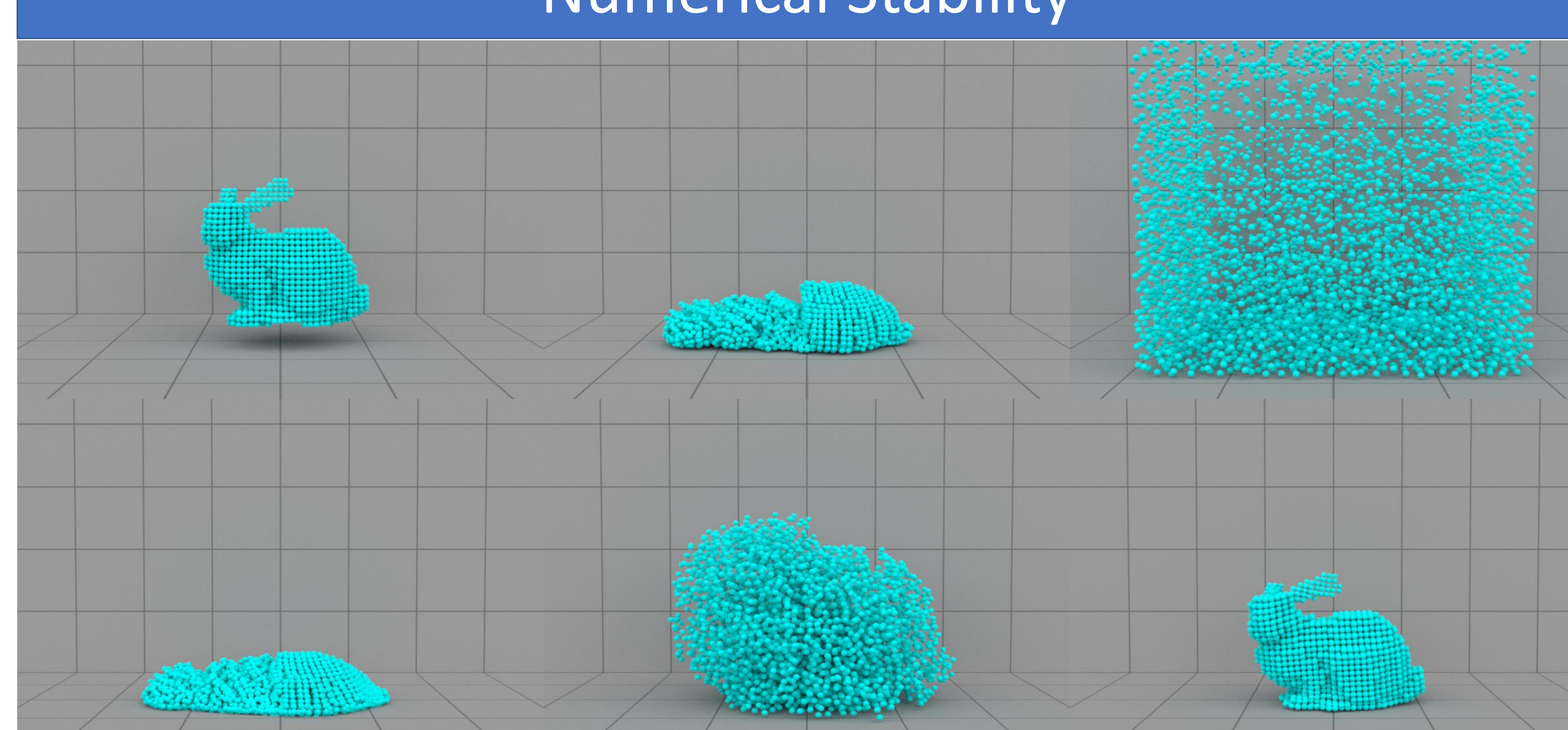
Buckling test. (Top) Laplacian form of viscosity with particle view (left) and mesh view (right). (Bottom) Full form of viscosity with particle view (left) and mesh view (right). Full form can generate buckling while Laplacian form fails.

Viscous Thread Coiling



Viscous thread coiling. Our method can generate complex coiling phenomenon.

Numerical Stability



Numerical stability test with different combinations of time steps and viscosities. (Top left) initial state. (Top middle) explicit integration with $\Delta t = 5.0\times10^{-6}$ s and $\mu = 1.0\times10^{3}$ kg/(s·m). (Top right) explicit integration with $\Delta t = 1.3\times10^{-3}$ s and $\mu = 1.0\times10^{3}$ kg/(s·m). (Bottom left) implicit integration with $\Delta t = 1.3\times10^{-3}$ s and $\mu = 1.0\times10^{3}$ kg/(s·m). (Bottom middle) explicit integration with $\Delta t = 5.0\times10^{-6}$ s and $\mu = 5.0\times10^{4}$ kg/(s·m). (Bottom right) implicit integration with $\Delta t = 1.0\times10^{-4}$ s and $\mu = 5.0\times10^{4}$ kg/(s·m).

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